## 16 Angles Formed by Chords, Secants, and Tangents

## I Inscribed Angles

D16-1 An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle. The inscribed angle may intercept a minor arc, a major arc or a semicircle.

T16-1 The measure of an inscribed angle is equal to half the measure of its intercepted arc.


C16-1 If two inscribed angles intercept the same arc, then the angles are congruent.
C16-2 If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.
C16-3 An angle inscribed in a semicircle is a right angle.


T16-2 Chord-Tangent Angle Theorem The measure of an angle formed by a chord and a tangent is equal to half the measure of the intercepted arc.

## Exercise I



1-6 Find $x, y, z$.
1.

2.

3.


4.

5.

6.

7. Prove: If two chords of a circle are parallel, the two arcs between the chords are congruent.
Given: $\overline{A B} \| \overline{C D}$.
Prove: $A C \cong B D$

8. Prove: $\triangle U X Z \sim \triangle Y V Z$.

9. Find the measures of the numbered angles in a regular polygon with 7 sides.

10. Given: $\overline{P T}$ is a tangent; $\overline{T U} \| \overline{P S}$. Find three similar triangles and prove them similar.


## II Other Angles

T16-3 Chord-Chord Angle Theorem The measure of an angle formed by two chords that intersect inside a circle is equal to half the sum of the measures of the intercepted arcs.

T16-4 Secant-Secant, Secant-Tangent, Tangent-Tangent Angle Theorem The measure of an angle formed by two secants, two tangents, or a secant and a tangent drawn from a point outside a circle is equal to half the difference of the measures of the intercepted arcs.

## Exercise II

1-9 Find the measure of each numbered angle.
1.


3.

4.


6.


8.

9.

10. $\overleftrightarrow{B Z}$ is tangent to $\odot O ; \overline{A C}$ is a diameter; $m B C=90$; $m C D=30 ; ~ m D E=20$.
Find the measure of each numbered angle.

11. (a) If $m R T=80$ and $m U S=40$, then $m \angle 1=$ $\qquad$ 2.
(b) If $m R U=130$ and $m T S=100$, then $m \angle 1=$ $\qquad$ .

(c) If $m \angle 1=50$ and $m R T=70$, then $m U S=$ $\qquad$ .
(d) If $m \angle 1=52$ and $m U S=36$, then $m R T=$ $\qquad$ .
12. $\overline{P X}$ and $\overline{P Y}$ are tangents.
(a) If $m X Z Y=250$, then $m \angle P=$ $\qquad$ .
(b) If $m X Y=90$, then $m \angle P=$ $\qquad$ .

(c) If $m \angle P=85$, then $m X Y=$ $\qquad$ .
13. $\overline{A T}$ is a tangent.
(a) If $m C T=110$ and $m B T=50$, then $m \angle A=$ $\qquad$ .
(b) If $m \angle A=40$ and $m B T=40$, then $m C T=$ $\qquad$ .

(c) If $m \angle A=35$ and $m C T=110$, then $m B T=$ $\qquad$ .
14. A Quadrilateral circumscribed about a circle has angles of $80^{\circ}, 90^{\circ}, 94^{\circ}$, and $96^{\circ}$. Find the measures of the four nonoverlapping arcs determined by the points of tangency.
15. Write an equation involving $a, b$, and $c$.

16. Find the ratio of $x: y$.


